Find the following limits.
Each answer should be a number, ∞, $-\infty$, or DNE (only if the other answers do not apply).

SCORE: ____ / 7 PTS

[a] $\lim_{x\to\infty} \dfrac{6}{2-7e^{-x}} = \dfrac{6}{2-0}$ (½)

$= 3$ (½)

[b] $\lim_{x\to-\infty} \dfrac{\sqrt{5x^6+1}}{17x^2-2x^3} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}}$ (1)

SINCE $x<0$, $\dfrac{1}{x^3} = -\sqrt{\dfrac{1}{x^6}}$

$= \lim_{x\to-\infty} \dfrac{-\sqrt{5+\frac{1}{x^6}}}{\frac{17}{x}-2}$ (1)

$= \dfrac{-\sqrt{5+0}}{0-2}$

$= \dfrac{\sqrt{5}}{2}$ (1)

[c] $\lim_{x\to 2} \dfrac{1+x}{4-2x}$ DNE (1)

$\lim_{x\to 2^+} \dfrac{1+x}{4-2x} = -\infty$ $\dfrac{3}{0^-}$ (½)

$\lim_{x\to 2^-} \dfrac{1+x}{4-2x} = \infty$ $\dfrac{3}{0^+}$ (½)

Let $f(x) = \dfrac{x}{1+2x}$.

SCORE: ____ / 8 PTS

[a] Find $f'(x)$.

$\lim_{h\to 0} \dfrac{\frac{x+h}{1+2(x+h)} - \frac{x}{1+2x}}{h}$

(1) POINT EACH

$= \lim_{h\to 0} \dfrac{(x+h)(1+2x) - x(1+2x+2h)}{h(1+2x+2h)(1+2x)}$

$= \lim_{h\to 0} \dfrac{h + 2hx - 2hx}{h(1+2x+2h)(1+2x)} = \dfrac{1}{(1+2x)^2}$

[b] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x = 3$.

$f(3) = \frac{3}{7}$ $f'(3) = \frac{1}{49}$

$y - \frac{3}{7} = \frac{1}{49}(x-3)$

[c] The position (in inches) of an object moving in a straight line is given by $s(t) = \dfrac{t}{1+2t}$, where t is the time in seconds.

Find the instantaneous velocity of the object at time $t = 4$. Give the correct units for your answer.

$s'(4) = \frac{1}{81}$ INCHES/SECOND

Determine if the function $f(x)=\begin{cases}1-x^3+x^6, & \text{if } x<-1\\ x-x^2+x^5, & \text{if } x\geq -1\end{cases}$ is continuous at $x=-1$. **SCORE: ____ / 4 PTS**

State your conclusion clearly, and justify using the definition of "continuous".
In addition, if it is not continuous, determine the type of discontinuity and justify using the appropriate definition.

(½) $\lim_{x\to -1^-}(1-x^3+x^6)=3$

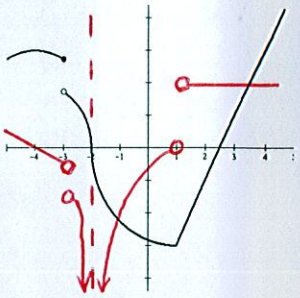
(½) $\lim_{x\to -1^+}(x-x^2+x^5)=-3$

} NOT EQUAL, SO $\lim_{x\to -1} f(x)$ DNE (1)

SO NOT CONTINUOUS (½)

(1) BOTH 1-SIDED LIMITS EXIST, BUT ARE NOT EQUAL, SO JUMP DISCONTINUITY (½)

The graph of $f(x)$ is shown on the right. **SCORE: ____ / 6 PTS**



[a] Find all x-coordinates where $f'(x)$ is undefined, and explain briefly why.

(1) $x=-3$ DISCONTINUITY,

(1) $x=-2$ VERTICAL TANGENT LINE,

(1) $x=1$ CUSP

MUST HAVE BOTH X-VALUE & REASON TO GET POINT

[b] Sketch a graph of $f'(x)$ on the same axes.

GRADED BY ME

Using complete sentences & proper mathematical notation, write the formal definition of "derivative (function)". **SCORE: ____ / 1 PT**

THE DERIVATIVE OF f IS $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

GRADED BY ME

Using complete sentences & proper mathematical notation, write the formal definition of "continuous (at a point)". **SCORE: ____ / 2 PTS**

f IS CONTINUOUS AT a IF $f(a)$ EXISTS,

$\lim_{x\to a} f(x)$ EXISTS

AND $\lim_{x\to a} f(x)=f(a)$

GRADED BY ME

The age at which you can retire depends on how much money you add to your retirement account each month. **SCORE: ____ / 2 PTS**
Suppose $r=f(a)$, where r is your retirement age (in years), and a is the amount of money you add to your retirement account each month (in thousands of dollars).

GRADED BY ME

[a] What does $f'(3)=-12$ mean ? Give the correct units for all numbers in your answer.

IF YOU ADD \$3000 TO YOUR RETIREMENT ACCOUNT EACH MONTH YOU COULD RETIRE 12 YEARS EARLIER FOR EACH ADDITIONAL \$1000 YOU ADDED EACH MONTH.

[b] Is there a value a_0 such that $f'(a_0)>0$? Why or why not ?

NO. IF YOU INCREASED YOUR MONTHLY DEPOSIT, YOUR RETIREMENT AGE WOULD ALWAYS DECREASE